6th International Symposium on Space Sailing (ISSS 2023), New York, USA, 5-9 June 2023

Solar Sail Torque Model Characterization for the Near Earth Asteroid Scout Mission

Benjamin DIEDRICH[a,*]

[a]*NASA MSFC/EV42, ESSCA/Axient Corporation, Huntsville, AL, USA*

Abstract

Near Earth Asteroid Scout (NEA Scout) was a mission to test solar sail propulsion for orbital transfer from cislunar space to flyby and image an asteroid. Had it succeeded, one of the mission goals was to characterize the solar torque on the sail to ensure successful attitude control for the orbit transfer and imaging the asteroid. The simulation used to develop the flight attitude control software uses the generalized model for solar sails, a tensor equation of the forces and torques on sails of arbitrary shape. Rios-Reyes and Scheeres developed a general process to update the torque tensor coefficients using estimates of sail torque over a range of directions to the sun. Their process was adapted and implemented for the specific case of NEA Scout using spacecraft telemetry collected during sail characterization maneuvers in combination with simulation models and parameters. The NEA Scout maneuvers were limited to the operating range of the mission and constraints of the control hardware and allowed safe testing of each attitude before proceeding to the next. The NEA Scout reaction wheel speeds are used to measure accumulated momentum, while the Active Mass Translator (AMT) position is used to subtract out the torque from the center of mass crossed with the sail force and isolate the torque from only the sail shape. The process was tested by running attitude control simulations of the characterization maneuvers, generating simulated telemetry, estimating the solar torques, then using a least squares estimating the solar torque coefficients using least-squares and then performing a least-squares fit to the solar torque tensor coefficients. These estimated coefficients were tested by evaluating the solar torques under the same conditions as the simulated telemetry and comparing to the true simulated torques. Solar force model updates can be performed separately by observing the effect of the sail on the trajectory, and the torque model can be refined using those solar force updates. This process met the needs of the NEA Scout mission and can be adapted to characterize the solar torque for other missions with different sails.

Keywords: solar sail, NEA Scout, torque, characterization, modeling, controls

Nomenclature

$\vec{\boldsymbol{M}}_{shape}$	Torque vector due to sail shape
$\vec{\boldsymbol{M}}_{force}$	Torque vector due to force and center of mass
$\vec{\boldsymbol{M}}_{total}$	Total torque acting on sail
$\vec{\boldsymbol{f}}_{sail}$	Sail force vector
$\vec{\boldsymbol{r}}_{cm}$	Body frame vector of spacecraft center of mass
p	Solar pressure
$a_{1,2,3}$	Derived optical coefficients
$\mathbf{K}^2$, $\mathbf{L}$	3x3 sail torque tensors
$\mathbf{K}^3$	3x3x3 sail torque tensor
$\hat{\boldsymbol{r}}_s$	Unit sun vector in the sail body frame
$\boldsymbol{K}$	Vector of unique torque tensor coefficients
Subscripts	
est	From estimates of torque coefficients
plant	From plant model torque coefficients
Δ	Difference between plant and estimated torque model
err	Relative error between plant and estimated torque model

1. Introduction

Solar sail spacecraft perform trajectory maneuvers by changing the direction of thrust generated by sunlight reflecting on the sail. For current sail designs where the sail is fixed to the spacecraft body, this requires controlling the attitude of the sailcraft relative to the incident sunlight over time to generate the required force profile for the mission. To design a working attitude control system, the sail torques need to be understood over the range of attitudes required for the mission to size the actuators and design the control software. Sail torque is driven by the three-dimensional

[*] Corresponding author, benjamin.l.diedrich@nasa.gov

shape of the thin sail membrane, and even small deviations from flatness can generate sizeable torques that the attitude control system needs to manage with margin. By contrast, the force on a sail is insensitive to small deviations from flatness. One of the goals of the Near Earth Asteroid Scout (NEA Scout) mission was to measure the solar torque on the sail and characterize the solar torque model used for simulating attitude control, to better understand how to control this mission in particular, and to understand sail torque more generally so that attitude control systems can be designed with confidence for future sail missions. This paper describes the process developed for NEA Scout to use spacecraft telemetry, mission data, and models to update the torque model. It is a practical application of the approach originally published by Rios-Reyes and Scheeres [1], who also developed the Generalized Model for Solar Sails [2] that was used to model the torque of the three-dimensional shape of the NEA Scout sail film.

This paper discusses the sail characterization concept of operation, flight and model data used in the analysis, sail characterization procedure, theory, simulation results, and finally the conclusions and future work. The simulation results show some success in solving for torque coefficients that predict the truth torque due to sail shape, despite not matching the coefficients well. There is plenty of room for improvement for the process. The absolute torque errors from the estimated coefficients are small, but the relative torque errors can be large in many cases.

2. Sail Characterization Concept of Operations

Had the NEA Scout mission succeeded, the planned mission was to deploy the sail in cislunar space after a few weeks of trajectory correction maneuvers with the cold gas reaction control system (RCS), followed by cislunar sailing, then an interplanetary phase to flyby an accessible asteroid. To ensure control of the sail for performing those maneuvers and to collect data for refining the sail torque model, torque characterization maneuvers were planned for a period of several days after sail deployment.

Characterization would have started with one day pointed directly at the sun (with an additional day for margin). Once stable control was demonstrated with sun pointing, the incidence would be increased by 10 deg increments, to 49 deg (1 deg short of the 50 deg operational limit for attitude control margin). At each sun incidence angle, the sail would be rolled to +/- 30. The roll is incremented in 10 deg increments at 20 deg SIA and above, because the roll changes slowly with roll at 10 deg SIA. Every change in sail attitude was planned to happen during a communications pass, so that if problems arose the spacecraft can be commanded to return to the previous safe attitude. The schedule is shown in Table 1.

Table 1. Sail characterization attitudes

SIA (deg)	Roll (deg)
0	0
10	0
10	+30
10	-30
20	-30
20	-20
20	-10
20	0
20	10
20	20
20	30
…	…
49	30
49	20
49	10
49	0
49	-10
49	-20
49	-30

While this does not cover the entire range of possible sun angles, and therefore provides incomplete data for full sail characterization, it does test the entire range of motion required for this mission's trajectory maneuvers. NEA Scout has a critical piece of control hardware called the Active Mass Translator (AMT) that shifts the center of mass to control the pitch & yaw solar torque to manage the momentum of the reaction wheels in two axes. Because of the volume constraints imposed by the 6U cubesat bus, the AMT has a limited range of motion in one axis and reduced control authority at larger roll angles.

3. Telemetry, Data, and Models

The telemetry required to characterize the NEA Scout sail includes:

- Time: reference all other telemetry and data
- Reaction wheel speeds: measure total torque as derivative of momentum
- Control mode (Hold or Slew): identify when spacecraft is holding attitude or moving
- AMT momentum management status: identify when the AMT has stopped moving
- Attitude estimate: used to compute the sun vector in the sail body frame
- AMT position: used to compute the torque caused by the sail force crossed with the center of mass offset

Additional mission data required for sail characterization includes:

- Ephemeris: used to compute the sun vector and distance to the sun
- Solar flux [6]: used to compute solar pressure at the time of measurements

Model data used for sail characterization includes the following parameters. Any updates to this information, like sail force characterization, will improve the estimates of the torque model.

- Reaction wheel alignment: project wheel speeds into body frame
- Reaction wheel inertia: convert wheel speeds to momentum
- AMT axis polarity: correct AMT position sign
- Sail force model: compute torque of force crossed with center of mass
- Mass property model: compute center of mass as function of AMT position

4. Sail Characterization Procedure

The general approach to sail torque model characterization is to estimate the total torque from measurements of the reaction wheel speeds while holding the sail attitude inertially fixed and solve for the sail torque coefficients that produce the observed torque. The details of this process are described below.

Once the telemetry, mission data, and model data are available, the sail characterization process can proceed as follows:

- Load model data
- Load trajectory ephemeris and solar flux
- Load sailcraft telemetry (flight or emulated)
- Split the telemetry into segments where the control mode is HOLD and the AMT has stopped moving so that the dynamics of slewing and AMT movement don't increase the uncertainty on the torque estimates

For each data segment:

- Compute reaction wheel momentum by multiplying wheel speed by wheel inertias
- Compute reaction wheel body frame momentum by multiplying alignment matrix by the per-wheel momentum
- Compute mean AMT position per segment (should be constant since the AMT shouldn't be moving)
- Fit polynomial to body frame wheel momentum
- Find body frame torque polynomial by taking derivative of momentum polynomial
- Fit polynomial to attitude estimate quaternions

For each data segment, pick sample times (start, middle, final times) to perform calibration calculations to smooth the data and reduce the number of data points used in the least-squares problem. For each data segment & sample time:

- Lookup sailcraft position using ephemeris
- Compute solar pressure using sailcraft position and solar flux
- Evaluate inertial to body quaternion polynomial
- Calculate sun vector in the body frame using inertial position and attitude quaternion
- Calculate center of mass using the mass properties model and AMT position
- Calculate the solar force using the sail force model, solar pressure, and body sun vector
- Calculate the solar torque from solar force by taking the cross product of the force with the center of mass
- Calculate the sail shape torque by subtracting the torque due to force from the reaction wheel torque estimate
- Normalize the sail shape torque by dividing by the solar pressure
- Compute the linear system (A-matrix) that maps a vector of the sail torque coefficients into normalized sail shape torque measurements (detailed in Theory section)

Once these steps are completed for all the data segments and sample times:

- Build the total least-squares A-matrix by vertically stacking all the A-matrices for each segment and sample
- Build the total least-squares b-vector by stacking the normalized estimates of sail shape torque
- Solve the least-squares problem for the vector of sail torque coefficients
- Rearrange the estimated sail torque coefficients into tensor form
- Test the estimated torque coefficients using them to calculate the torque at each sample time using the corresponding sun vector and solar pressure; compare to the estimates and plant model values if available

5. Theory

The general least-squares problem is formulated as the solution $\boldsymbol{x}$ to the equation:

$$\boldsymbol{A}\,\boldsymbol{x} = \boldsymbol{b} \tag{1}$$

where $\boldsymbol{b}$ is a vector of observations, and $\boldsymbol{A}$ is a linearized model (i.e. matrix) that maps the parameters $\boldsymbol{x}$ to the observations of the system in question.

In the case of the sail torque characterization, this problem is formulated from the generalized sail torque model equation from [2]. For NEA Scout, no assumptions on symmetry were made, so a full set of 36 unique torque coefficients are solved for. With assumptions on symmetry, a smaller set of coefficients can be solved for. The tensor equation for the sail shape torque is:

$$\vec{\boldsymbol{M}}_{\text{shape}} = p(a_2\mathbf{K}^2 \cdot \hat{\boldsymbol{r}}_s - a_1\hat{\boldsymbol{r}}_s \cdot \mathbf{K}^3 \cdot \hat{\boldsymbol{r}}_s - a_3\hat{\boldsymbol{r}}_s \cdot \mathbf{L} \cdot \hat{\boldsymbol{r}}_s) \tag{2}$$

The sail characterization process estimates the coefficients of the rank-2 $\mathbf{K}^2$ and $\mathbf{L}$ tensors, and the rank-3 $\mathbf{K}^3$ tensor, for a total of 36 unique coefficients. The only symmetry is in the 2nd and 3rd indices of $\mathbf{K}^3$. The parameters a1, a2, and a3 are reformulations of the flat-plate optical coefficients r, s, B_b, B_f, e_b, and e_f:

$$\begin{aligned} a_1 &= 2rs \\ a_2 &= \frac{B_f(1-s)r+(1-r)(e_fB_f-e_bB_b)}{e_f+e_b} \\ a_3 &= 1-rs \end{aligned} \tag{3}$$

The tensor products were implemented in Matlab by reformulating them as matrix multiplications. $\mathbf{K}^2$ and $\mathbf{L}$ are represented as 3x3 matrices *K2* and *L*, and $\mathbf{K}^3$ as a 3x3x3 array *K3*. They were multiplied with the sun vector $\hat{\mathbf{r}}_s$ (represented as column vector *rsun*) using the following expressions:

$$\mathbf{K}^2 \cdot \hat{\boldsymbol{r}}_s \equiv \mathrm{K2} * \mathrm{rsun} \tag{4}$$

$$\hat{\boldsymbol{r}}_s \cdot \mathbf{K}^3 \cdot \hat{\boldsymbol{r}}_s \equiv \mathrm{reshape}(\mathrm{reshape}(\mathrm{K3},9,3) * \mathrm{rsun}, 3,3) * \mathrm{rsun} \tag{5}$$

$$\hat{\boldsymbol{r}}_s \cdot \mathbf{L} \cdot \hat{\boldsymbol{r}}_s \equiv (\mathrm{rsun}'{*}\mathrm{L}){*}\mathrm{skew}(\mathrm{rsun}) \tag{6}$$

Where:

$$\mathrm{skew}(\mathrm{sun}) \equiv \begin{bmatrix} 0 & -\mathrm{rsun}(3) & \mathrm{rsun}(2) \\ \mathrm{rsun}(3) & 0 & -\mathrm{rsun}(1) \\ -\mathrm{rsun}(2) & \mathrm{rsun}(1) & 0 \end{bmatrix} \tag{7}$$

The Matlab implementation was used in the plant model of the simulation and in post-processing to test the estimated moment coefficients. It was also implemented in the computer algebra system Maxima to formulate the least-squares problem.

The sail torque equation was expressed as a symbolic function of the solar pressure, optical coefficients, sun vector components, and individual torque coefficients that returned a 3D torque vector.

The tensors K2a, K3a, and La were defined in Maxima as follows, so that they can be symbolically evaluated:

```
K2a : genmatrix(lambda([i,j],arraymake(K2,[i,j])),3,3);
K3a : [
genmatrix(lambda([i,j],
   arraymake(K3, cons(i,sort([j,1])))),3,3),
genmatrix(lambda([i,j],
   arraymake(K3,cons(i,sort([j,2])))),3,3),
genmatrix(lambda([i,j],
   arraymake(K3,cons(i,sort([j,3])))),3,3)];
La : genmatrix(lambda([i,j],arraymake(L,[i,j])),3,3);
```

The shape torque is defined as a Maxima function. The full expansion of all the coefficients is too long to express here, but it can be expanded using this function:

```
shape_torque(p,r,a,K2,K3,L) :=
   p * (a[2] * K2 . r
   - a[1] * addcol(K3[1] . r, K3[2] . r, K3[3] . r) . r
   - a[3] * transpose((transpose(rv) . L) . skew(r)));
```

The $\hat{\mathbf{r}}_s$ vector and *a* coefficients are defined symbolically in Maxima so the shape torque can be evaluated:

```
rv : genmatrix(lambda([i,j],arraymake(r,[i])), 3, 1);
ac : [a[1], a[2], a[3]];
```

With this, the sail shape torque can be symbolically evaluated with the following Maxima expression. The full expansion is too long to show here.

```
M_shape :
   expand(shape_torque(p, rv, av, K2a, K3a, La));
```

The moment equation is then reformulated into the linear form $\boldsymbol{A}\,\boldsymbol{x} = \boldsymbol{b}$ where the $\boldsymbol{x}$ being solved for is a vector of all 36 unique moment coefficients and $\boldsymbol{b}$ is the sail torque. The vector of torque coefficients is named $\boldsymbol{K}$ and is built from K2, K3, and L by listing their coefficients in order by index and removing redundant K3 coefficients (those with the same 2nd and 3rd indices). This results in the vector:

$$\boldsymbol{K} = \begin{matrix} [K_{1,1}^2\ K_{1,2}^2\ K_{1,3}^2\ K_{2,1}^2\ K_{2,2}^2\ K_{2,3}^2\ K_{3,1}^2\ K_{3,2}^2\ K_{3,3}^2 \dots \\ K_{1,1,1}^3\ K_{1,1,2}^3\ K_{1,1,3}^3\ K_{1,2,2}^3\ K_{1,2,3}^3\ K_{1,3,3}^3 \dots \\ K_{2,1,1}^3\ K_{2,1,2}^3\ K_{2,1,3}^3\ K_{2,2,2}^3\ K_{2,2,3}^3\ K_{2,3,3}^3 \dots \\ K_{3,1,1}^3\ K_{3,1,2}^3\ K_{3,1,3}^3\ K_{3,2,2}^3\ K_{3,2,3}^3\ K_{3,3,3}^3 \dots \\ L_{1,1}\ L_{1,2}\ L_{1,3}\ L_{2,1}\ L_{2,2}\ L_{2,3}\ L_{3,1}\ L_{3,2}\ L_{3,3}]^T \end{matrix} \tag{7}$$

This operation was done in Maxima by processing the arrays K2, K3, and L then processing it using the code:

```
K : transpose(matrix(
delete(0,append(unique(apply(append,args(K2a))),
unique(apply(append,apply(append,map(args,K3a)))),
unique(apply(append,args(La)))))));
```

Next, the A-matrix is solved for from the vector of coefficients $\boldsymbol{K}$ and vector of torques $\overrightarrow{\boldsymbol{M}}_{shape}$. Using Maxima, the following two functions are used. The first is a modified version of the 'isolate' function that factors out the K-vector coefficients from each term of the shape torque M_shape, without using intermediate expressions:

```
betterisolate1(expr,x) :=
 (iso : isolate(expr,x),
  if atom(iso) then 0 else
   (if subst(0,x,second(iso)) = 0
      then subst(1,x,second(iso))
      else subst(1,x,first(iso))));
```

This function is then used to create the entire A-matrix for the three dimensions of the shape torque and 36 K-vector coefficients:

```
make_A_matrix(M, K) :=
   genmatrix(lambda([i,j],
      betterisolate1(M[i,1], K[j,1])), 3, length(K));
Am : make_A_matrix(M_shape, K);
```

The A-matrix then satisfies the equation:

$$\overrightarrow{\boldsymbol{M}}_{shape} = \mathbf{A} \cdot \boldsymbol{K} \tag{8}$$

The transpose of the A-matrix with solar pressure divided out for a sail with no symmetry and all 36 torque coefficients is shown in Equation 9.

The torque due to the sail force and center of mass offset caused by the AMT is:

$$\overrightarrow{\boldsymbol{M}}_{force} = -\vec{\mathbf{r}}_{\mathrm{cm}} \times \vec{\boldsymbol{f}}_{sail} \tag{10}$$

After the total torque on the sail is found from the derivative of the reaction wheel momentum, the estimate of the shape torque is found by subtracting the torque from the sail force.

$$\overrightarrow{\boldsymbol{M}}_{shape} = \overrightarrow{\boldsymbol{M}}_{total} - \overrightarrow{\boldsymbol{M}}_{force} \tag{11}$$

For each measurement, one shape torque is calculated, and all the individual shape torques are stacked vertically to form the complete least-squares $\boldsymbol{b}$ vector. An **A** matrix is also calculated for each observation, and these are stacked vertically into the complete **A** matrix. These are input to the least-squares solver to find the vector of torque coefficients, **K**. The **K** coefficients are reformulated into their corresponding tensors.

$$\frac{\mathbf{A}^{\mathrm{T}}}{p} = \begin{bmatrix} r_1 a_2 & 0 & 0 \\ a_2 r_2 & 0 & 0 \\ a_2 r_3 & 0 & 0 \\ 0 & r_1 a_2 & 0 \\ 0 & a_2 r_2 & 0 \\ 0 & a_2 r_3 & 0 \\ 0 & 0 & r_1 a_2 \\ 0 & 0 & a_2 r_2 \\ 0 & 0 & a_2 r_3 \\ -a_1 r_1 & 0 & 0 \\ -2a_1 r_1 r_2 & 0 & 0 \\ -2a_1 r_1 r_3 & 0 & 0 \\ -a_1 r_2 & 0 & 0 \\ -2a_1 r_2 r_3 & 0 & 0 \\ -a_1 r_3 & 0 & 0 \\ 0 & -a_1 r_1 & 0 \\ 0 & -2a_1 r_1 r_2 & 0 \\ 0 & -2a_1 r_1 r_3 & 0 \\ 0 & -a_1 r_2 & 0 \\ 0 & -2a_1 r_2 r_3 & 0 \\ 0 & -a_1 r_3 & 0 \\ 0 & 0 & -a_1 r_1 \\ 0 & 0 & -2a_1 r_1 r_2 \\ 0 & 0 & -2a_1 r_1 r_3 \\ 0 & 0 & -a_1 r_2 \\ 0 & 0 & -2a_1 r_2 r_3 \\ 0 & 0 & -a_1 r_3 \\ 0 & r_1 a_3 r_3 & -r_1 r_2 a_3 \\ -r_1 a_3 r_3 & 0 & r_1\ a_3 \\ r_1 r_2 a_3 & -r_1\ a_3 & 0 \\ 0 & r_2 a_3 r_3 & -r_2\ a_3 \\ -r_2 a_3 r_3 & 0 & r_1 r_2 a_3 \\ r_2\ a_3 & -r_1 r_2 a_3 & 0 \\ 0 & a_3 r_3 & -r_2 a_3 r_3 \\ -a_3 r_3 & 0 & r_1 a_3 r_3 \\ r_2 a_3 r_3 & -r_1 a_3 r_3 & 0 \end{bmatrix} \tag{9}$$

6. Results

The first set of results was generated for the planned NEA Scout sail characterization attitudes. A total of 33 simulations were run for each characterization attitude, as described in section 2. The first part of each simulation started with the AMT at position [0, 0], followed by a momentum management activation where the AMT converged on the non-zero equilibrium position for that attitude. Each simulation runs for 3 hours to generated simulated telemetry for processing.

The characterization process starts by fitting a 2nd order polynomial to the wheel momentum, normalized by solar pressure. A few examples of this data are shown in Fig. 1, collected at different SIA and roll angles. The gaps in the data are when the AMT activates to reduce the RW momentum and trim out the pitch/yaw torques. Data is only used when the AMT isn't moving and spacecraft is holding an inertial attitude to make the data smoother so a polynomial and its derivative can be easily calculated.

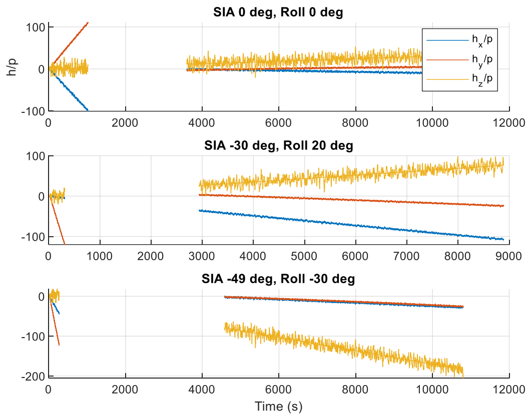

Fig. 1. Reaction wheel momentum / solar pressure

The total torque normalized by solar pressure is found by taking the polynomial derivative of the normalized momentum. These are 1st order polynomials to account for gradual changes in torque over the telemetry segment from effects like the movement of the sail relative to the sun direction. To use it in the characterization process without excessive data points, the torque polynomial is evaluated at the beginning, midpoint, and end of each continuous telemetry segment. Examples of these are shown in Fig. 2.

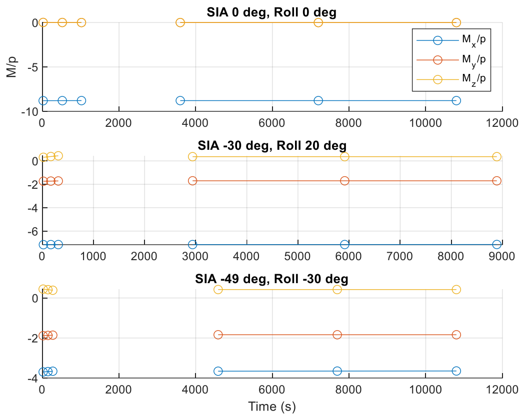

Fig. 2. Total solar torque / solar pressure

Once the total normalized torque is calculated, measurement of the sail shape torque is found by subtracting the torque from the solar force crossed with the CM. The CM is calculated from the mass model using the AMT position as an input. The sail force is calculated from a sail model, that can itself be updated by observing the sail's effect on the trajectory. This normalized torque, due only to sail shape is show in Fig. 3.

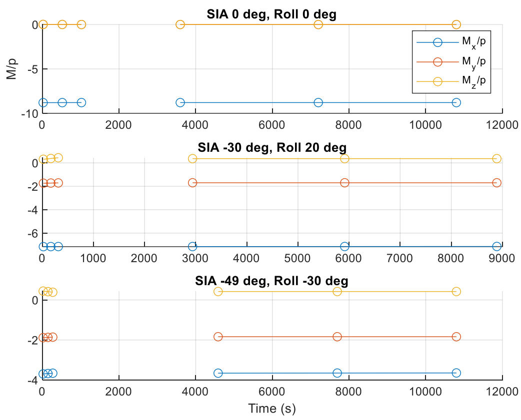

Fig. 3. Sail shape torque / solar pressure

This is the data needed for building the ***b*** vector in the least-squares problem. All these normalized shape torque vectors are stacked vertically into the total ***b*** vector. The sail body frame sun vector is calculated at times corresponding to the normalized shape torques from the inertial-to-body quaternions and position of the sun from the sail orbit and sun ephemeris. The unit sun vector components and optical coefficients are used to compute an ***A*** matrix according to Equation 9 for each normalized shape torque, and stacked vertically into the total ***A*** matrix.

Then, the least-squares problem is solved for the vector of shape coefficients (Equation 7). These are reformulated back into the tensors in Equation 2. These tensors are then compared to those in the truth model. Unfortunately, with the NEA Scout simulation and attitude schedule, the torque coefficients do not clearly match the plant model. To demonstrate this, the **L** tensors are shown in Equations 12 and 13. The $\mathbf{K}^2$ and $\mathbf{K}^3$ tensors have similar differences.

$$\mathbf{L}_{est} = \begin{bmatrix} -0.0805 & -0.0122 & 0.0030 \\ 0.0035 & -0.0453 & 0.0043 \\ 0.0022 & 0.6433 & 0.1258 \end{bmatrix} \quad (12)$$

$$\mathbf{L}_{plant} = \begin{bmatrix} -0.2476 & -0.0009 & -0.0012 \\ -0.0025 & 0.2328 & -0.0050 \\ 0.0002 & 4.9006 & 18.2043 \end{bmatrix} \quad (13)$$

The reasons for the poor match are not clearly known, but some possibilities include the limited range of SIA and roll angle to accommodate NEA Scout operations, and the large number of coefficients being searched for (36) relative to the number of observations (33 attitudes with ~6 data points each).

Next, the estimated coefficients were used to compare the torques they calculate against the plant model torques. Using the same model inputs – sun vector, solar flux, solar distance – the torque from the sail shape was calculated using the estimated sail torque coefficients and compared to the same torque calculation using the plant model coefficients. Equation 14 is the difference between the torque calculations. It is plotted for all 219 samples (73 telemetry sets with 3 samples each) in Fig. 4. All the torques computed with the estimated coefficients come in below 10x10^{-8} Nm of the plant model.

$$\vec{\boldsymbol{M}}_{\text{shape},\Delta} = \vec{\boldsymbol{M}}_{shape,plant} - \vec{\boldsymbol{M}}_{shape,est} \quad (14)$$

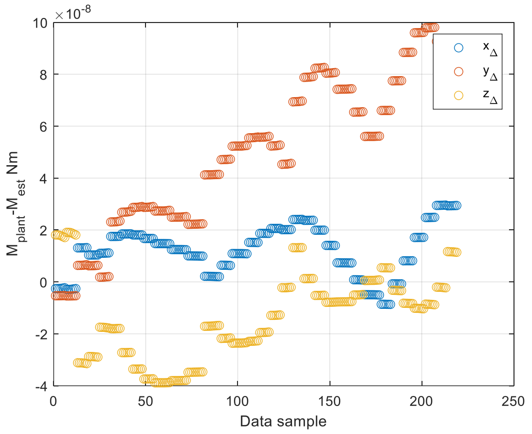

Fig. 4. Torque difference between plant model and estimated torque coefficients

Next, the error factor relative to the modeled plant moment was calculated according to Equation 15 and plotted in Fig. 5, both at full scale and zoomed in to within a factor of 0.1. The differences are much more significant, with a factor of ~150 for the worst data point. The relative error is much better for many of the data points. For the y-axis, many are within a factor of +/- 0.05. For the x-axis many are within +/- 0.1. The roll axis is less accurate, within a factor of +/- 1.0.

$$\vec{\boldsymbol{M}}_{shape,err} = \frac{\vec{\boldsymbol{M}}_{shape,plant} - \vec{\boldsymbol{M}}_{shape,est}}{\vec{\boldsymbol{M}}_{shape,plant}} \quad (15)$$

From the computation of the differences in sail torque, the absolute torque accuracy is bounded. In some cases, the relative error is quite large. The reasons for this are likely because the torque estimates are small and within the noise floor of the measurements. The AMT deliberately minimizes the total momentum and torque observable on the wheels, and the torque estimate is then driven by the model of the sail force crossed with the center of mass, rather than measurements. The torque differences plotted in Fig. 4 may represent a noise floor, below which the torque estimates will have large relative error. If both the torque measured by the wheels and modeled by the force and CM are within the that order of magnitude, they may be expected to have large errors.

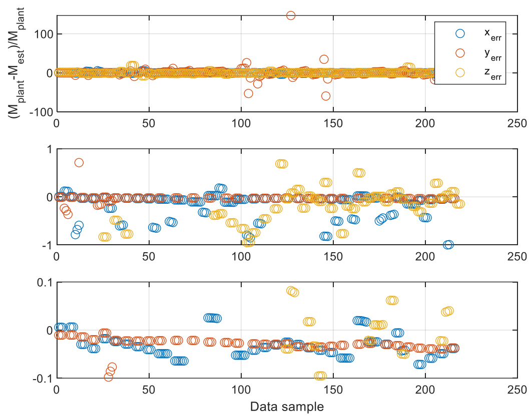

Fig. 5. Relative torque error from plant model and estimated torque coefficients

7. Conclusions and Future Work

The process described can solve for estimated torque coefficients of the sail that are able to replicate the plant model torques, although with large relative errors on the roll axis and with several large outlier torque errors. The results suggest that the process can be improved to reproduce the true torques of a sail more accurately.

Possible improvements to be examined include:

- Rejection of measurements that fall below the noise floor

- Enlarged range of angles to characterize the sail over for missions without the attitude constraints of the NEA Scout AMT, like the cloverleaf pattern studied by Rios-Reyes and Scheeres [1]
- Simplified set of moment coefficients that include symmetries to reduce the number of parameters being solved for
- Analyze noise sources of the different axes in more detail and how to mitigate them
- Tolerance of control system design to errors in torque model

This process is planned to be adapted to other solar sail missions with different attitude control hardware and mission scenarios, including Solar Cruiser [7] and the Advanced Composite Solar Sail System (ACS3) [8]. Solar Cruiser's mission to the sun-Earth L1 Lagrange region allows plenty of time to perform similar characterization operations as NEA Scout and has an AMT design that doesn't limit the range of roll angles. ACS3 will fly in Earth orbit where additional environmental torques (aerodynamic, gravity gradient, Earth radiation pressure) will need to be removed.

Acknowledgments

The author would like to acknowledge his fellow team members on the Near Earth Asteroid Scout and Solar Cruiser guidance & control teams. Their thorough job of developing the control software and models of those missions enabled this work to happen.

References

[1] L. Rios-Reyes and D. J. Scheeres, Solar-Sail Navigation: Estimation of Force, Moments, and Optical Parameters, Journal of Guidance, Control, and Dynamics Vol. 30 No. 3 (2007) 660-668.

[2] L. Rios-Reyes and D. J. Scheeres, Generalized Model for Solar Sails, Journal of Spacecraft and Rockets Vol. 42 No. 1 (2005) 182-185.

[3] Juan Orphee, Ben Diedrich, Brandon Stiltner, Chris Becker, and Andrew Heaton, Solar Sail Attitude Control System for the NASA Near Earth Asteroid Scout Mission, International Symposium on Solar Sailing 2017, Kyoto, Japan.

[4] Matlab, https://www.mathworks.com/products/matlab.html, Accessed: April 18 2023.

[5] Maxima: A Computer Algebra System, https://maxima.sourceforge.io/, Accessed: April 18 2023.

[6] TSIS-1: Total Solar Irradiance Data, https://lasp.colorado.edu/tsis/data/tsi-data/ Accessed: April 19 2023.

[7] L. Johnson and others. The NASA Solar Cruiser Mission – Solar Sail Propulsion Enabling Heliophysics Missions. Logan, Utah, USA, 2022. 36th Annual Small Satellite Conference. Paper SSC22-II-03.

[8] K. Wilkie. The NASA Advanced Composite Solar Sail System (ACS3) Flight Demonstration: A Technology Pathfinder for Practical Smallsat Solar Sailing. Logan, Utah, USA, 2021. 35th Annual Small Satellite Conference. Paper SSC21-II-10.